

Radion as a possible dark matter candidate

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Abstract

The discrepancy between observed virial and baryonic mass in galaxy clusters have lead to the missing mass problem. To resolve this, a new, non-baryonic matter field, known as dark matter has been invoked. However, till date no possible constituents of the dark matter components are known. This has led to various models, by modifying gravity at large distances to explain the missing mass problem. The modification to gravity appears very naturally when effective field theory on a lower dimensional manifold, embedded in a higher dimensional spacetime is considered. It has been shown that in a scenario with two lower dimensional manifolds separated by a finite distance is capable to address the missing mass problem, which in turn determines the kinematics of the brane separation.

1 Introduction

Recent astrophysical observations strongly suggest existence of non-baryonic dark matter at the galactic as well as extra-galactic scales (if the dark matter is baryonic in nature, the third peak in the Cosmic Microwave Background power spectrum would have been lower compared to the observed height of the spectrum [1]). These observations can be divided into two branches — (a) behavior of galactic rotation curves and (b) mass discrepancy in clusters of galaxies [2].

The first one, i.e., rotation curves of spiral galaxies, show clear evidences of problems associated with Newtonian and general relativity prescriptions [2–4]. In these galaxies neutral hydrogen clouds are observed much beyond the extent of luminous Baryonic matter. In Newtonian description, the equilibrium of these clouds moving in a circular orbit of radius r is obtained through equality of centrifugal and gravitational force. For cloud velocity $v(r)$, the centrifugal force is given by v^2/r and the gravitational force by $GM(r)/r^2$, where $M(r)$ stands for total gravitational mass within radius r . Equating these two will lead to the mass profile of the galaxy

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as, $M(r) = rv^2/G$. This immediately posed serious problem, for at large distances from the center of the galaxy, the velocity remains nearly constant $v \sim 200$ km/s, which suggests that mass inside radius r should increase monotonically with r , even though at large distance very little luminous matter can be detected [2–4].

The mass discrepancy of galaxy clusters also provides direct hint for existence of dark matter. The mass of galaxy clusters, which are the largest virialized structures in the universe, can be determined in two possible ways — (i) from the knowledge about motion of the member galaxies one can estimate the virial mass M_V , secondly, (ii) estimating mass of individual galaxies and then summing over them in order to obtain total baryonic mass M . Almost without any exception M_V turns out to be much large compared to M , typically one has $M_V/M \sim 20 - 30$ [2–4]. Recently, new methods have been developed to determine the mass of galaxy clusters, these are — (i) dynamical analysis of hot X-ray emitting gas [5] and (ii) gravitational lensing of background galaxies [6] — these methods also lead to similar results. Thus dynamical mass of galaxy clusters are *always* found to be in excess compared to their visible or baryonic mass. This missing mass issue can be explained through postulating that, every galaxy and galaxy cluster is embedded in a halo made up of dark matter. Thus the difference $M_V - M$ is originating from the mass of the dark matter halo, the galaxy cluster is embedded in.

The physical properties and possible candidates for dark matter can be summarized as follows: dark matter is assumed to be non-relativistic (hence cold and pressure-less), interacting only through gravity. Among many others, the most popular choice being weakly interacting massive particles. Among different models, the one with sterile neutrinos (with masses of several keV) has attracted much attention [7, 8]. Despite of few successes it comes with its own limitations. In the sterile neutrino scenario the X-ray produced from their decay can enhance production of molecular hydrogen and thereby speeding up cooling of gas and early star formation [9]. Even after a decade long experimental and observational efforts no non-gravitational signature for the dark matter has ever been found. Thus *a priori* the possibility of breaking down of gravitational theories at galactic scale cannot be excluded [10–16].

A possible and viable way to modify the behaviour of gravity in our four dimensional space-time is by introducing extra spatial dimensions. The extra dimensions were first introduced to explain the hierarchy problem (i.e., observed large difference between the weak and Planck energy scales) [17–19]. However the initial works did not incorporate gravity, but used large extra dimensions (and hence large volume factor) to reduce the Planck scale to TeV scale. Introduction of gravity, i.e., warped extra dimensions drastically altered the situation. In [20] it was first shown that anti-de Sitter solution in higher dimensional spacetime (henceforth referred to as bulk) leads to exponential suppression of the energy scales on the visible four dimensional embedded sub-manifold (called as brane) thereby solving the hierarchy problem. Even though this scenario of warped geometry model solves the hierarchy problem, it also introduces additional correction terms to the gravitational field equations, leading to deviations from Einstein’s theory at high energy, with interesting cosmological and black hole physics applications [21–29]. This conclusion is not bound to Einstein’s gravity alone but holds in higher curvature gravity theories¹ as well [28, 29, 37]. Since the gravitational field equations get modified due to intro-

¹In addition to introduction of extra dimensions we could also modify the gravity theory without invoking ghosts, which uniquely fixes the gravitational Lagrangian to be Lanczos-Lovelock Lagrangian. These Lagrangians have special thermodynamic properties and also modifies behaviour of four-dimensional gravity [30–36]. However in this work we shall confine ourselves exclusively within the framework of Einstein gravity and shall try to explain the missing mass problem from kinematics of the radion field.

duction of extra dimensions it is legitimate to ask, whether it can solve the problem of missing mass in galaxy clusters. Several works in this direction exist and can explain the velocity profile of galaxy clusters. However they emerge through the following setup:

- obtaining effective gravitational field equations on a lower dimensional hypersurface, starting from the full bulk spacetime, which involves additional contributions from the bulk Weyl tensor. The bulk Weyl tensor in spherically symmetric systems leads to a component behaving as mass and is known as “dark mass” (we should emphasize that this notion extends beyond Einstein’s gravity and holds for any arbitrary dimensional reduction [28, 29, 37]). It has been shown in [38] that introduction of the “dark mass” term is capable to yield an effect similar to the dark matter. Some related aspects were also explored in [39–42], keeping the conclusions unchanged.
- In the second approach, the bulk spacetime is always taken to be anti-De Sitter such that bulk Weyl tensor vanishes. Unlike the previous case, which required S^1/Z_2 orbifold symmetry, arbitrary embedding has been considered in [43] following [44]. This again introduces additional corrections to the gravitational field equations. These additional correction terms in turn lead to the observed virial mass for galaxy clusters.

However all these approaches were valid for a single brane system. In this work we generalize previous results for a two brane system. This approach not only gives a handle on the hierarchy problem at the level of Planck scale but is also capable of explaining the missing mass problem at the scale of galaxy clusters. Moreover, in this setup the additional corrections will depend on the radion field (for a comprehensive discussion see [25]), which represents the separation between the two branes. Hence in our setup the missing mass problem for galaxy clusters can also shed some light on the kinematics of the separation between the two branes.

The paper is organized as follows — In Section 2, after providing a brief review of the setup we have derived effective gravitational field equations on the visible brane which will involve additional correction terms originating from the radion field to modify the gravitational field equations. In Section 3 we have explored the connection of the radion field, dark matter and the velocity profile of galaxy clusters using relativistic Boltzmann equations. Finally, we conclude with a discussion on our results.

Throughout our analysis, we have set the fundamental constant c to unity. All the Greek indices μ, ν, α, \dots run over the brane coordinates. We will also use the standard signature $(- + + \dots)$ for the spacetime metric.

2 Effective Gravitational Field Equations on the Brane

The most promising candidate for getting effective gravitational field equations on the brane originates from Gauss-Codazzi equation. However these equations are valid on a lower dimensional hypersurface (i.e., on the brane) embedded in a higher dimensional bulk. Hence this works only for a single brane system. But, the brane world model, addressing hierarchy problem requires existence of two branes, where the above method is not applicable. To tackle the problem of two brane system we need to invoke the radion field (i.e., separation between two branes), which has significant role in the effective gravitational field equations. The effective field equations on the visible brane (i.e., the brane on which Planck scale is exponentially suppressed) in this scenario

corresponds to,

$${}^{(4)}G_\nu^\mu = \frac{\kappa^2}{\ell} \frac{1}{\Phi} T_\nu^{(\text{vis})\mu} + \frac{\kappa^2}{\ell} \frac{(1+\Phi)^2}{\Phi} T_\nu^{(\text{hid})\mu} + \frac{1}{\Phi} (D^\mu D_\nu \Phi - \delta_\nu^\mu D^2 \Phi) + \frac{\omega(\Phi)}{\Phi^2} \left[D^\mu \Phi D_\nu \Phi - \frac{1}{2} \delta_\nu^\mu (D\Phi)^2 \right] \quad (1)$$

where ℓ is the bulk curvature radius and κ^2 stands for bulk gravitational constant. In the above expression D stands for the four-dimensional covariant derivative, $g_{\mu\nu}$ being the induced metric on the visible brane. Also, $D^2 \Phi$ stands for $D^\mu D_\mu \Phi$ and $(D\Phi)^2 = D_\mu \Phi D^\mu \Phi$. Among the energy momentum tensors, $T_{\mu\nu}^{\text{hid}}$ is the one on the hidden brane and $T_{\mu\nu}^{\text{vis}}$ corresponds to that on the visible brane. The scalar field $\Phi(x)$ appearing in the above effective equation is directly connected to the radion field $d(x)$ (representing separation between the branes) such that $\omega(\Phi)$ and Φ has the following expressions

$$\Phi = \exp\left(\frac{2d}{\ell}\right) - 1; \quad \omega(\Phi) = -\frac{3}{2} \frac{\Phi}{1+\Phi} \quad (2)$$

We will assume $d(x)$, the brane separation to be finite and everywhere nonzero. This suggests that $\Phi(x)$ should always be greater than zero and shall never become infinity. Finally we also have a differential equation satisfied by Φ , which can be written as

$$D_\mu D^\mu \Phi = \frac{\kappa^2}{\ell} \frac{1}{2\omega + 3} \left(T^{(\text{vis})} + T^{(\text{hid})} \right) - \frac{1}{2\omega + 3} \frac{d\omega}{d\Phi} D_\mu \Phi D^\mu \Phi \quad (3)$$

where $\omega(\Phi)$ has been defined in Eq. (2) and T^{vis} and T^{hid} stands for the trace of energy momentum tensor on the hidden and visible branes respectively. We are mainly interested in spherically symmetric spacetime, in which generically the line element takes the following form:

$$ds^2 = -e^\nu dt^2 + e^\lambda dr^2 + r^2 d\Omega^2 \quad (4)$$

This particular form of the metric is used extensively in various physical contexts, for example in obtaining black hole solution, particle orbit, perihelion precession of planetary orbits, bending of light and in various other astrophysical phenomenon [45–47]. Given this metric ansatz we can compute all the derivatives of the scalar field and being a static situation, the brane separation is assumed to depend on radial coordinate only. Thus we will only have terms involving derivative with respect to r (these will be denoted by prime). First we can rewrite the scalar field equation, which will be a differential equation for Φ . We will also assume that there is no matter on the hidden brane, but only on the visible brane, which is assumed to be perfect fluid. Thus on the visible brane we have energy momentum tensor to be, $T_\mu^{\nu(\text{vis})} = \text{diag}(-\rho, p, p_\perp, p_\perp)$, with the trace being given by, $T = -\rho + p + 2p_\perp$. From now on we will remove the label ‘vis’ from the energy momentum tensor, since only on the visible brane energy momentum tensor is non-zero. With these inputs and the above spherically symmetric metric ansatz we obtain the scalar field equation as,

$$\partial_r^2 \Phi + \frac{2}{r} \partial_r \Phi + \left(\frac{\nu' - \lambda'}{2} \right) \partial_r \Phi = \frac{\kappa^2}{\ell} \frac{1+\Phi}{3} T^{(\text{vis})} e^\lambda + \frac{1}{2(1+\Phi)} (\partial_r \Phi)^2 \quad (5)$$

Having derived the scalar field equation, next we need to obtain the field equations for gravity with the metric ansatz given by Eq. (4). These will be differential equations for $\nu(r)$ and $\mu(r)$ respectively. We can separate out the time-time component, radial component and transverse components leading to

$$-e^{-\lambda} \left(\frac{1}{r^2} - \frac{\lambda'}{r} \right) + \frac{1}{r^2} = \frac{\kappa^2}{\ell} \frac{\rho + \rho_0}{\Phi} + e^{-\lambda} \frac{\nu'}{2} \frac{\Phi'}{\Phi} + \frac{\kappa^2}{\ell} \frac{1 + \Phi}{3\Phi} T - \frac{e^{-\lambda} \Phi'^2}{4\Phi(1 + \Phi)} \quad (6)$$

$$e^{-\lambda} \left(\frac{\nu'}{r} + \frac{1}{r^2} \right) - \frac{1}{r^2} = \frac{\kappa^2}{\ell} \frac{p - \rho_0}{\Phi} - \frac{2}{r} e^{-\lambda} \frac{\Phi'}{\Phi} - e^{-\lambda} \frac{\nu'}{2} \frac{\Phi'}{\Phi} - \frac{3}{4} \frac{\Phi'^2}{\Phi(1 + \Phi)} e^{-\lambda} \quad (7)$$

$$e^{-\lambda} \left(\nu'' + \frac{\nu'^2}{2} + \frac{\nu' - \lambda'}{r} - \frac{\nu' \lambda'}{2} \right) = \frac{\kappa^2}{\ell \Phi} 2(p_{\perp} - \rho_0) + \frac{2}{r} e^{-\lambda} \frac{\Phi'}{\Phi} - \frac{2\kappa^2}{3\ell} \frac{1 + \Phi}{\Phi} T + \frac{1}{2} \frac{e^{-\lambda} \Phi'^2}{\Phi(1 + \Phi)} \quad (8)$$

where primes denote derivatives with respect to radial coordinate. In the above field equations along with the perfect fluid, we have contributions from the brane cosmological constant. Here we have inserted a brane energy density ρ_0 , where ρ_0 and brane cosmological constant is related via $\rho_0 = \Lambda/8\pi G$. Here G is the four dimensional gravitational constant. Finally, we have contribution from the radion field itself, since it appears on the right hand side of gravitational field equations. Having derived the field equations we will now proceed to determine the effect of the radion field on the kinematics of galaxy clusters and hence its implications for the missing mass problem.

3 Virial Theorem in Galaxy Clusters, Kinematics of The Radion Field and Dark Matter

It is well known that the galaxy clusters are the largest virialized systems in the universe [2], on top of which we will assume them to be isolated, spherically symmetric system such that the metric of the spacetime region they are situated in can be presented by the metric ansatz presented in Eq. (4). Galaxies within the galaxy cluster are treated as identical, point particles satisfying general relativistic collision less Boltzmann equation.

To get the Boltzmann equation we first need to setup the phase space of which one-particle phase spaces are the most important. A one-particle state with mass m is described by its position $x \in \mathcal{M}$, where \mathcal{M} stands for the spacetime manifold and its four-momentum $p \in T_x$, where T_x is the tangent space at x . Hence the one particle phase space corresponds to the tangent bundle $\mathcal{M} \times T_x$, such that,

$$P_{\text{phase}} := \{(x, p) \mid x \in \mathcal{M}, p \in T_x, p^2 = -m^2\} \quad (9)$$

We are now in a position to define the distribution function $f(x, p)$ of a multi-particle system, such that it is continuous, non-negative and describe a state of the system. The distribution function is defined on P_{phase} , yielding the number dN of the particles of the system, within a volume dV located at x and have four-momentum p within a three surface element $d\vec{p}$ in momentum space. All the observables can be constructed out of various moments of the distribution function.

Now let $\{x^a\}$ be a local coordinate system defined in the manifold \mathcal{M} . Among these coordinates ∂_t is timelike and future directed, while ∂_α is spacelike, such that $\{\partial/\partial x^a\}$ stands for a

natural basis of tangent vectors. Since four-momentum $p \in T_x$, we have, $p = p^a(\partial/\partial x^a)$. This suggests to define a local coordinates in the phase space where $\{z^A\}$ denotes a local coordinate system in P_{phase} , such that $z^a = x^a$ and $z^{a+4} = p^a$. Hence A runs over $0, 1, \dots, 7$. Hence a natural basis at a point in the phase space is $\{\partial/\partial z^A\} = \{\partial/\partial x^a, \partial/\partial p^a\}$.

Using all these ingredients and following [38] the transport equation for the propagation of a particle in a curved Riemannian spacetime is given by the Boltzmann equation,

$$\left(p^a \frac{\partial}{\partial x^a} - p^a p^b \Gamma_{ab}^i \frac{\partial}{\partial p^i} \right) f = 0 \quad (10)$$

It is appropriate to introduce tetrad fields $e_a^{(\mu)}$, where μ runs over internal space indices and satisfies the equation $e_a^{(\mu)} e^{(\nu)a} = \eta^{(\mu)(\nu)}$, independent of the spacetime point. For the spherically symmetric line element as in Eq. (4) the following tetrads would be helpful,

$$e_a^{(0)} = e^{\nu/2} \delta_a^0; \quad e_a^{(1)} = e^{\lambda/2} \delta_a^1; \quad e_a^{(2)} = r \delta_a^2; \quad e_a^{(3)} = r \sin \theta \delta_a^3 \quad (11)$$

Let u^a be the normalized four-velocity of a galaxy within the galaxy cluster, such that $u_a u^a = -1$. Then tetrad components of u^a are $u^{(\mu)} = e_a^{(\mu)} u^a$. The relativistic Boltzmann equation in tetrad form reads,

$$\left(u^{(\mu)} e_{(\mu)}^a \frac{\partial}{\partial x^a} + u^{(\mu)} u^{(\nu)} \gamma_{(\mu)(\nu)}^{(\alpha)} \frac{\partial}{\partial u^{(\alpha)}} \right) f = 0 \quad (12)$$

which can be obtained from Eq. (10) by substituting $p_a = m u_a$ and $u^a = u^{(\beta)} e_{(\beta)}^a$ respectively. Also we have $\gamma_{(\beta)(\mu)}^{(\alpha)} = e_{(\beta)}^b e_{(\mu)}^c \nabla_c e_b^{(\alpha)}$ as the Ricci rotation coefficients. Now we introduce $u^{(0)} = u_t$, $u^{(1)} = u_r$, $u^{(2)} = u_\theta$ and $u^{(3)} = u_\phi$ with the assumption that distribution function depends on radial coordinate only (which is justified given the fact that the geometry is spherically symmetric). Finally Eq. (12) reduces to the following form [38],

$$\begin{aligned} u_r \frac{\partial f}{\partial r} - \left(\frac{1}{2} u_t^2 \frac{\partial \nu}{\partial r} - \frac{u_\theta^2 + u_\phi^2}{r} \right) \frac{\partial f}{\partial u_r} - \frac{1}{r} u_r \left(u_\theta \frac{\partial f}{\partial u_\theta} + u_\phi \frac{\partial f}{\partial u_\phi} \right) \\ - \frac{1}{r} e^{\lambda/2} u_\phi \cot \theta \left(u_\theta \frac{\partial f}{\partial u_\phi} - u_\phi \frac{\partial f}{\partial u_\theta} \right) = 0 \end{aligned} \quad (13)$$

However the coefficient of $\cot \theta$ must vanish due to spherical symmetry. Hence the distribution function can be a function of r , u_r and $u_\theta^2 + u_\phi^2$ only. Now multiplying by $m u_r du$, where m stands for galaxy mass and du is the velocity space element. Assuming that distribution function vanishes rapidly as velocities tend to large values we find that after integrating over the cluster [38],

$$- \int_0^R 4\pi \rho [\langle u_r^2 \rangle + \langle u_\theta^2 \rangle + \langle u_\phi^2 \rangle] r^2 dr + \frac{1}{2} \int_0^R 4\pi r^3 \rho [\langle u_t^2 \rangle + \langle u_r^2 \rangle] \frac{\partial \nu}{\partial r} dr = 0 \quad (14)$$

where R stands for the radius of the galaxy cluster. Using the distribution function, the energy momentum tensor of the matter turns out to be,

$$T_{ab} = \int f m u_a u_b du \quad (15)$$

which leads to the following expressions for energy density and pressure as,

$$\rho_{\text{eff}} = \rho \langle u_t^2 \rangle; \quad p_{\text{eff}}^{(r)} = \rho \langle u_r^2 \rangle; \quad p_{\text{eff}}^{(\perp)} = \rho \langle u_\theta^2 \rangle = \rho \langle u_\phi^2 \rangle \quad (16)$$

Having discussed the basics of the relativistic Boltzmann equation we have obtained corresponding expressions for the energy density and pressure. It is now time to include gravity to the picture. For that we use these expressions for energy density and pressure in the gravitational field equations presented in Eq. (6), Eq. (7) and Eq. (8). Finally adding all of them together we arrived at

$$\begin{aligned} e^{-\lambda} \left(\nu'' + 2 \frac{\nu'}{r} + \frac{\nu'^2}{2} - \frac{\nu' \lambda'}{2} \right) &= \frac{\kappa^2}{\ell \Phi} \left(\rho_{\text{eff}} + p_{\text{eff}}^{(r)} + 2p_{\text{eff}}^{(\perp)} \right) - \frac{1}{2} \frac{e^{-\lambda} \Phi'^2}{\Phi(1+\Phi)} \\ &\quad - \frac{\kappa^2}{3\ell} \frac{1+\Phi}{\Phi} \left(-\rho_{\text{eff}} + p_{\text{eff}}^{(r)} + p_{\text{eff}}^{(\perp)} - 4\rho_0 \right) - \frac{\kappa^2}{\ell \Phi} \rho_0 \end{aligned} \quad (17)$$

To obtain Eq. (17), we have used the expression for trace of the energy momentum tensor. We also remember that ρ_0 stands for vacuum energy density. At this stage it is useful to introduce certain assumptions, since actually we are interested in a post-Newtonian formulation of the effective gravitational field equations. The two assumptions are — (a) ν and λ are small so that any quadratic expressions constructed out of them can be neglected in comparison to the linear one. Secondly, (b) the velocity of the galaxies are assumed to be much smaller compared to the velocity of light, which suggests, $\langle u_r^2 \rangle, \langle u_\theta^2 \rangle, \langle u_\phi^2 \rangle \ll \langle u_t^2 \rangle$. This in turn implies $\rho_{\text{eff}} \gg p_{\text{eff}}^{(r)}, p_{\text{eff}}^{(\perp)}$ such that all the pressure terms can be neglected in comparison to the energy density. Applying all these approximation schemes Eq. (17) can be rewritten as,

$$\frac{1}{2r^2} \frac{\partial}{\partial r} (r^2 \nu') = \frac{\kappa^2}{6\ell} \rho + \frac{2\kappa^2}{3\ell} \rho_0 - \frac{1}{4} \frac{\Phi'^2}{\Phi(1+\Phi)} + \frac{2\kappa^2 \rho}{3\ell \Phi} - \frac{\kappa^2 \rho_0}{3\ell \Phi} \quad (18)$$

We can also perform the same schemes of approximation to Eq. (14), which leads to,

$$-2K + \frac{1}{2} \int_0^R 4\pi r^3 \rho \nu' dr = 0 \quad (19)$$

where K stands for the total kinetic energy of the galaxies within the galaxy cluster and has the following expression,

$$K = \int_0^R dr \, 4\pi r^2 \rho \left[\frac{1}{2} \{ \langle u_r^2 \rangle + \langle u_\theta^2 \rangle + \langle u_\phi^2 \rangle \} \right] \quad (20)$$

The mass within a small volume of radial extent dr has the expression $dM(r) = 4\pi r^2 \rho dr$, where in this and subsequent expressions ρ will indicate $\rho(r)$. Thus total mass of the system can be given by integral of $dM(r)$ over the full size of the galaxy. The main contribution comes from mass of intra-cluster gas and stars along with other particles, e.g., massive neutrinos. We can also define the gravitational potential energy Ω of the cluster as,

$$\Omega = - \int_0^R \frac{GM(r)}{r} dM(r) \quad (21)$$

Finally multiplying [Eq. \(18\)](#) by r^2 and integrating from 0 to r , we arrive at,

$$\frac{1}{2}r^2\frac{\partial\nu}{\partial r} = \frac{\kappa^2}{6\ell}\int_0^r r^2\rho(r)dr + \frac{2\kappa^2\rho_0}{3\ell}\int_0^r r^2dr + \frac{\kappa^2}{4\pi\ell}M_\Phi(r) \quad (22)$$

where we have defined:

$$M_\Phi(r) = \int_0^r dr 4\pi r^2 \left(-\frac{\ell}{4\kappa^2} \frac{\Phi'^2}{\Phi(1+\Phi)} + \frac{2\rho}{3\Phi} - \frac{\rho_0}{3\Phi} \right) \quad (23)$$

This object captures all the effect of the radion field on the gravitational mass distribution of galaxy clusters and thus may be called as the “radion mass”. Also the total baryonic mass of the galaxy cluster within a radius r can be obtained by integrating the energy density over the size of the galaxy cluster, which leads to, $M(r) = 4\pi\int_0^r r^2\rho(r)dr$, using which we finally arrive at the following form for [Eq. \(22\)](#):

$$\frac{1}{2}r^2\frac{\partial\nu}{\partial r} = \frac{\kappa^2}{6\ell}\frac{M(r)}{4\pi} + \frac{2\kappa^2\Lambda}{3\ell}\frac{r^3}{3} + \frac{\kappa^2}{4\pi\ell}M_\Phi(r) \quad (24)$$

Earlier we have defined the gravitational potential associated with M , the baryonic mass. We can define an identical object using the radion mass as well, leading to an potential term Ω_Φ . Given the potentials we can introduce three radius — (a) R_V , the virial radius, obtained using total baryonic potential and baryonic mass, (b) R_I , the inertia radius, obtained from moment of inertia of the galaxy cluster and finally (c) R_Φ , the radion radius obtained from the radion mass (for detailed expressions see [Appendix A](#)). Using these expressions and the definition for virial mass, $M_V = \sqrt{2KR_V/G}$, yields the following expression,

$$\frac{M_V}{M} = \sqrt{\frac{\kappa^2}{24\pi G\ell} + \frac{2\kappa^2\rho_0}{9\ell G}\frac{R_V R_I^2}{M} + \frac{\kappa^2}{4\pi G\ell}\frac{R_V}{R_\Phi}\frac{M_\Phi^2}{M^2}} \quad (25)$$

For most of the clusters, the virial mass M_V is three times compared to the baryonic mass M and thus for all practical purposes the first term inside the square root, which is of order unity can be neglected with respect to the other two. The second term yields the contribution from the brane cosmological constant, which is several orders of magnitude smaller compared to the observed mass and thus can also be neglected. Finally, the virial mass turns out to be,

$$\frac{M_V}{M} \approx \frac{M_\Phi}{M} \sqrt{\frac{\kappa^2}{4\pi G\ell}\frac{R_V}{R_\Phi}} \quad (26)$$

Among the various terms in the above expression, virial mass M_V is determined from the study of velocity dispersion of galaxies within the cluster and is much large than the visible mass. The above expression shows that if the radion field kinematics is such that M_{tot} is equal to M_Φ . Then that in turn will lead to the correct virial mass of the galaxy clusters. The effect of radion field and hence of extra dimension can also be probed through gravitational lensing.

To see that, let us explore the differential equation for Φ , which has not yet been considered. Solving that will lead to some leading order behaviour of the radion field Φ , which in turn would affect M_Φ . Thus crucial thing is whether M_Φ behaves as r at large distance from the core of the

cluster. In which case from the above equation, we readily observe that the galaxy virial mass would also scale as $M_V \sim r$ explaining the issue of dark matter and galaxy rotation curve. To answer all these let us start by using the differential equation for Φ . There we will work under same approximation schemes, i.e., will be neglecting all the quadratic terms, e.g., $\nu'\Phi'$, Φ'^2 , will set $e^\lambda \sim 1$ and shall neglect vacuum energy contribution ρ_0 to obtain (for general expression see [Appendix A](#)),

$$\Phi'' + \frac{2}{r}\Phi' = -\frac{\kappa^2}{3\ell}(1 + \Phi)\rho \quad (27)$$

Multiplying both sides by r^2 and integrating twice we obtain (noting that Φ'^2 should not contribute)

$$\Phi = -\frac{\kappa^2}{12\pi\ell} \int dr \frac{M(r)}{r^2} \quad (28)$$

Here $M(r)$ stands for the mass of the baryonic matter within radius r and we know from observations that the density of the baryonic matter falls as $\rho_c(r_c/r)^{3\beta}$, where $\beta > 1$ and r_c stands for the core radius of the cluster. Thus it is straightforward to compute the mass profile, which goes as $\sim r^{3-3\beta}$, except for some constant contribution. Hence finally after integration we obtain the radion field to vary with the radial distance as $r^{1-3\beta}$. However note that the mass of the radion field, i.e., M_Φ under these approximations (matter is non-relativistic and field is weak) can be obtained as,

$$M_\Phi(r) = \int_0^r dr 4\pi r^2 \frac{2\rho}{3\Phi} = 24\pi(3\beta - 2)(\beta - 1)\frac{\ell}{\kappa^2}r \quad (29)$$

Thus the radion mass indeed scales linearly with radial distance which would correctly reproduce the observed virial mass of the galaxy cluster. Also its velocity profile does not die down at large r as expected. Thus the radion field kinematics can explain the kinematics of the galaxy cluster very well and thus the missing mass problem can be described without invoking any additional matter component. Thus an extra spatial dimension leads to a radion field, which can produce an elegant explanation for the dark matter in galaxy clusters.

4 Discussion

Brane world models can address some of the long standing puzzles in theoretical physics, namely — (a) the hierarchy problem and (b) the cosmological constant problem. To solve the hierarchy problem we need two branes, with warped five dimensional geometry such that energy scale on the visible brane gets suppressed exponentially leading to TeV scale physics. For the cosmological constant brane tension plays a crucial role. Two brane models naturally inherit an additional field, the separation between the branes (known as the radion field). Radion field is also very important in both macroscopic and microscopic physics, for it can have possible signatures in inflationary scenario [23–25], black hole physics [48, 49], collider searches [50], etc. Along with the hierarchy and cosmological constant problem there is another very important problem in physics, which is called the missing mass problem. This appears since baryonic and virial mass of a galaxy cluster do not coincide. In this work using a two brane setup we have shown that,

along with gauge hierarchy and cosmological constant problem, this model is also capable of solving the missing mass problem through the kinematics of the brane separation, i.e., radion field. Due to the presence of this additional field, the gravitational field equations on the brane gets modified and yields additional correction terms on top of Einstein's field equations. By considering relativistic Boltzmann equation we have derived the virial mass of galaxy clusters, which depends on an effective additional mass constructed out of radion field. Moreover these correction terms modifies the structure of gravity and hence the motion under its influence at large distance, thereby producing a linear increase in the virial mass of the galaxy clusters. This in turn leads to the appropriate velocity law for galaxies within a galaxy cluster, solving the missing mass problem.

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A Appendix: Derivations of Various Expressions Used in Text

In this appendix, we summarize derivations of important expressions presented in the main text. We hope this will be helpful to clarify the important algebraic steps to arrive at various results in the main body of this paper.

We start with various derivatives of the scalar field Φ in this spherically symmetric coordinate:

$$\begin{aligned} D^2\Phi &= \frac{1}{\sqrt{-g}}\partial_\mu (\sqrt{-g}g^{\mu\nu}\partial_\nu\Phi) \\ &= \frac{1}{\exp[(\nu+\lambda)/2]r^2\sin\theta}\partial_r [\exp[(\nu-\lambda)/2]r^2\partial_r\Phi] \\ &= e^{-\lambda}\partial_r^2\Phi + \frac{2}{r}e^{-\lambda}\partial_r\Phi + e^{-\lambda}\left(\frac{\nu'-\lambda'}{2}\right)\partial_r\Phi \end{aligned} \quad (30)$$

$$(D\Phi)^2 = g^{rr}\partial_r\Phi\partial_r\Phi = e^{-\lambda}(\partial_r\Phi)^2 \quad (31)$$

$$D_r D^r\Phi = g^{rr}(\partial_r^2\Phi - \Gamma_{rr}^r\partial_r\Phi) = e^{-\lambda}\left(\partial_r^2\Phi - \frac{\lambda'}{2}\partial_r\Phi\right) \quad (32)$$

$$D^t D_t\Phi = -e^{-\lambda}\frac{\nu'}{2}\partial_r\Phi \quad (33)$$

$$D^\theta D_\theta\Phi = D^\phi D_\phi\Phi = \frac{1}{r}e^{-\lambda}\partial_r\Phi \quad (34)$$

Using which the field equation for Φ takes the following form:

$$D_\mu D^\mu\Phi = \frac{\kappa^2}{\ell}\frac{1+\Phi}{3}T^{(\text{vis})} + \frac{1}{2(1+\Phi)}D_\mu\Phi D^\mu\Phi \quad (35)$$

Let us now turn our attention to the gravitational field equations. We will start with the temporal component such that:

$$\begin{aligned}
G_t^t &= \frac{\kappa^2}{\ell} \frac{1}{\Phi} T_t^t + \frac{1}{\Phi} (D_t D^t \Phi - D^2 \Phi) + \frac{3}{4} \frac{1}{\Phi(1+\Phi)} (D\Phi)^2 \\
&= -\frac{\kappa^2}{\ell} \frac{\rho + \Lambda}{\Phi} - e^{-\lambda} \frac{\nu'}{2} \frac{\partial_r \Phi}{\Phi} - \frac{\kappa^2}{\ell} \frac{1+\Phi}{3\Phi} T - \frac{1}{2\Phi(1+\Phi)} (D\Phi)^2 + \frac{3}{4} \frac{1}{\Phi(1+\Phi)} (D\Phi)^2 \\
&= -\frac{\kappa^2}{\ell} \frac{\rho + \Lambda}{\Phi} - e^{-\lambda} \frac{\nu'}{2} \frac{\Phi'}{\Phi} - \frac{\kappa^2}{\ell} \frac{1+\Phi}{3\Phi} T + \frac{e^{-\lambda} \Phi'^2}{4\Phi(1+\Phi)}
\end{aligned} \tag{36}$$

Then the radial component:

$$\begin{aligned}
G_r^r &= \frac{\kappa^2}{\ell} \frac{1}{\Phi} T_r^r + \frac{1}{\Phi} (D_r D^r \Phi - D^2 \Phi) - \frac{3}{2} \frac{1}{\Phi(1+\Phi)} D^r \Phi D_r \Phi + \frac{3}{4} \frac{1}{\Phi(1+\Phi)} (D\Phi)^2 \\
&= \frac{\kappa^2}{\ell} \frac{p - \Lambda}{\Phi} + e^{-\lambda} \left(\frac{\Phi''}{\Phi} - \frac{\lambda'}{2} \frac{\Phi'}{\Phi} \right) - e^{-\lambda} \frac{\Phi''}{\Phi} - \frac{2}{r} e^{-\lambda} \frac{\Phi'}{\Phi} - e^{-\lambda} \left(\frac{\nu' - \lambda'}{2} \right) \frac{\Phi'}{\Phi} \\
&\quad - \frac{3}{2} \frac{\Phi'^2}{\Phi(1+\Phi)} e^{-\lambda} + \frac{3}{4} \frac{\Phi'^2}{\Phi(1+\Phi)} e^{-\lambda} \\
&= \frac{\kappa^2}{\ell} \frac{p - \Lambda}{\Phi} - \frac{2}{r} e^{-\lambda} \frac{\Phi'}{\Phi} - e^{-\lambda} \frac{\nu'}{2} \frac{\Phi'}{\Phi} - \frac{3}{4} \frac{\Phi'^2}{\Phi(1+\Phi)} e^{-\lambda}
\end{aligned} \tag{37}$$

and finally the transverse part yields:

$$\begin{aligned}
G_\theta^\theta &= G_\phi^\phi = \frac{\kappa^2}{\ell\Phi} T_\theta^\theta + \frac{1}{\Phi} (D_\theta D^\theta \Phi - D^2 \Phi) + \frac{3}{4} \frac{1}{\Phi(1+\Phi)} (D\Phi)^2 \\
&= \frac{\kappa^2}{\ell\Phi} (p_\perp - \Lambda) + \frac{1}{r} e^{-\lambda} \frac{\Phi'}{\Phi} - \frac{\kappa^2}{3\ell} \frac{1+\Phi}{\Phi} T - \frac{1}{2} \frac{1}{\Phi(1+\Phi)} (D\Phi)^2 + \frac{3}{4} \frac{1}{\Phi(1+\Phi)} (D\Phi)^2 \\
&= \frac{\kappa^2}{\ell\Phi} (p_\perp - \Lambda) + \frac{1}{r} e^{-\lambda} \frac{\Phi'}{\Phi} - \frac{\kappa^2}{3\ell} \frac{1+\Phi}{\Phi} T + \frac{1}{4} \frac{e^{-\lambda} \Phi'^2}{\Phi(1+\Phi)}
\end{aligned} \tag{38}$$

Addition of these three equations and assuming the system to be non-relativistic, [Eq. \(17\)](#) leads to,

$$\begin{aligned}
\frac{\nu''}{2} + \frac{\nu'}{r} &= \frac{\kappa^2}{2\ell\Phi} \rho - \frac{1}{4} \frac{\Phi'^2}{\Phi(1+\Phi)} - \frac{\kappa^2}{6\ell} \frac{1+\Phi}{\Phi} (-\rho - 4\Lambda) - \frac{\kappa^2}{\ell\Phi} \Lambda \\
&= \frac{\kappa^2}{6\ell} \frac{4+\Phi}{\Phi} \rho + \frac{\kappa^2}{3\ell} \frac{2\Phi-1}{\Phi} \Lambda - \frac{1}{4} \frac{\Phi'^2}{\Phi(1+\Phi)} \\
&= \frac{\kappa^2}{6\ell} \rho + \frac{2\kappa^2}{3\ell} \Lambda - \frac{1}{4} \frac{\Phi'^2}{\Phi(1+\Phi)} + \frac{2\kappa^2 \rho}{3\Phi\ell} - \frac{\kappa^2 \Lambda}{3\ell\Phi}
\end{aligned} \tag{39}$$

Multiplying [Eq. \(22\)](#) with $4\pi r \rho(r)$ and integrating we obtain:

$$\begin{aligned}
\frac{1}{2} \int 4\pi \rho(r) r^3 \nu' dr &= \frac{\kappa^2}{24\pi\ell} \int 4\pi \rho(r) r M(r) dr + \frac{2\kappa^2 \Lambda}{3\ell} \int 4\pi \rho(r) r \frac{r^3}{3} dr + \frac{\kappa^2}{4\pi\ell} \int 4\pi \rho(r) r M_\Phi dr \\
&= \frac{\kappa^2}{24\pi\ell} \int \frac{M(r)}{r} dM(r) + \frac{2\kappa^2 \Lambda}{9\ell} \int r^2 dM(r) + \frac{\kappa^2}{4\pi\ell} \int \frac{M_\Phi}{r} dM(r)
\end{aligned} \tag{40}$$

These are the expressions used in main text. We also need to define the following objects:

$$\Omega = - \int \frac{GM}{r} dM; \quad \Omega_\Phi = - \int \frac{GM_\Phi}{r} dM; \quad I = \int r^2 dM \quad (41)$$

Then Eq. (40) takes the following form:

$$2K + \frac{\kappa^2}{24\pi G\ell} \Omega + \frac{\kappa^2}{4\pi G\ell} \Omega_\Phi - \frac{2\kappa^2 \Lambda}{9\ell} I = 0 \quad (42)$$

Let us introduce three radius R_V , R_I and R_Φ as:

$$R_V = \frac{M^2}{\int \frac{M}{r} dM}; \quad R_I^2 = \frac{\int r^2 dM}{M}; \quad R_\Phi = \frac{M_\Phi^2}{\int \frac{M_\Phi}{r} dM} \quad (43)$$

Then the above defined objects, namely, Ω , I and Ω_Φ reduces to,

$$\Omega = -\frac{GM^2}{R_V}; \quad I = MR_I^2; \quad \Omega_\Phi = -\frac{GM_\Phi^2}{R_\Phi} \quad (44)$$

On using these relations in the expression for Kinetic energy K , finally leads to

$$\frac{GM_V^2}{R_V} + \frac{\kappa^2}{24\pi G\ell} \left(-\frac{GM^2}{R_V} \right) - \frac{2\kappa^2 \Lambda}{9\ell} (MR_I^2) + \frac{\kappa^2}{4\pi G\ell} \left(-\frac{GM_\Phi^2}{R_\Phi} \right) = 0 \quad (45)$$

The above expression on rearrangement leads to Eq. (25). Now we need to solve for Φ , which satisfies the following differential equation,

$$\Phi'' + \frac{2}{r} \Phi' + \left(\frac{\nu' - \lambda'}{2} \right) \Phi' = \frac{\kappa^2}{3\ell} (1 + \Phi) (-\rho - 4\rho_0) + \frac{\Phi'^2}{2(1 + \Phi)} \quad (46)$$

which under integration leads to,

$$r^2 \frac{\Phi'}{1 + \Phi} = -\frac{\kappa^2}{3\ell} \int r^2 \rho(r) dr = -\frac{\kappa^2}{12\pi\ell} M(r) \quad (47)$$

Integrating once again we will immediately obtain Eq. (28).

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